





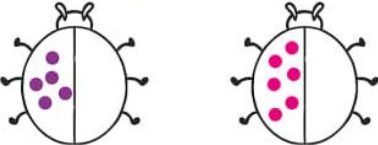
# Calculation policy: Multiplication

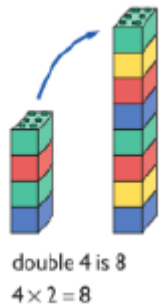
Key language: double, groups of, lots of, equal groups, repeated addition, times, multiplied by, product, factor, multiplier, multiplicand

$$\begin{array}{c} 6 \times 3 = 18 \\ \text{Factor} \quad \text{Factor} \quad \text{Product} \\ \text{(or Multiplier)} \quad \text{(or Multiplicand)} \end{array}$$

EYFS

- They solve problems including doubling

Concrete	Pictorial	Abstract
<p>Use a range of familiar objects to understand the concept of doubling, e.g. socks, hands, gloves, fingers, cubes, lego, numicon, dominos</p>  	<p>Paint on half a butterfly and fold it to see doubling.</p>  <p>Draw pictures to show doubles.</p>   <p>Double 5 is _____</p> <p>Double 6 is _____</p>	<p>Singing</p> <p>Recall doubles</p>

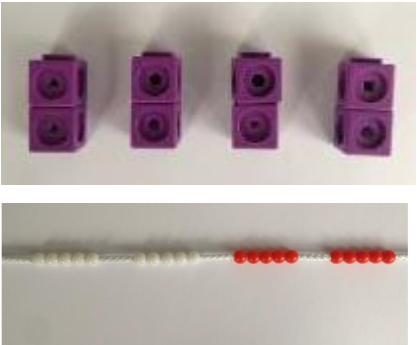
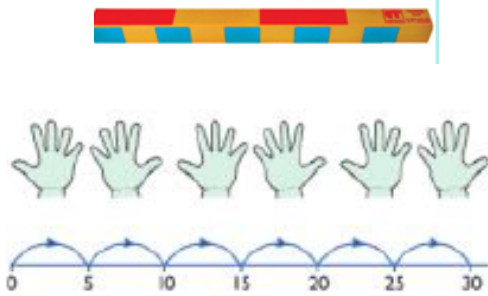



Draw pictures to show how to double a number.  
Double 4 is 8



Band 1

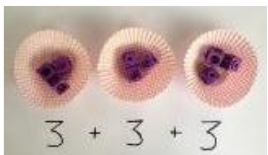
- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- count in 2s, 5s and 10s

Concrete	Pictorial	Abstract
<p><b>Counting in multiples</b> Count in multiples supported by concrete objects in equal groups.</p> 	<p>Use a counting stick, number line or pictures to continue support in counting in multiples.</p>  <p>Splat the multiples on a hundred square</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> <p>Count with hidden groups of 2, 5 and 10.</p> 

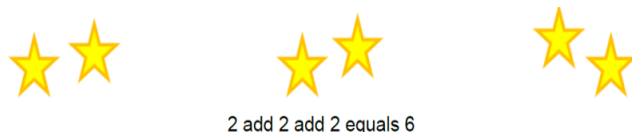
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Repeated addition**

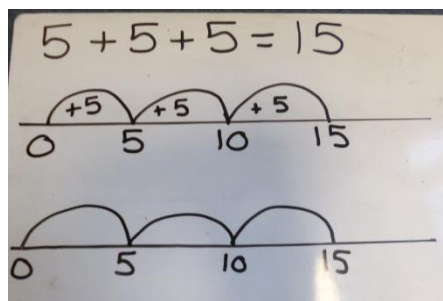
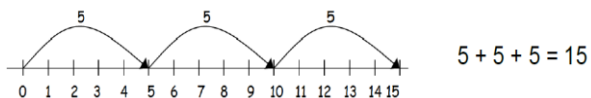
Children use different objects to add equal groups



There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



$5 + 5 + 5 = 15$



Write addition sentences to describe objects and pictures.



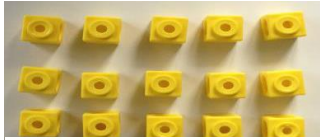
**Create arrays** using children/counters/ cubes to show repeated addition as '....rows of...' or ...'lots of...' and commutativity.

Draw arrays in different rotations to find **commutative** multiplication sentences.

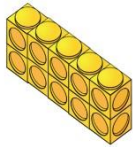
Children to be able to use an array to write a range of calculations and understand the meaning of the equals sign e.g.



4 rows of 5 is 20  
5 rows of 4 is 20

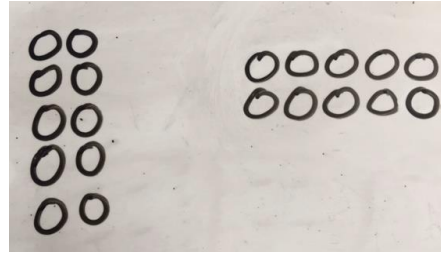


2 lots of 5



5 lots of 2


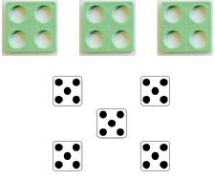
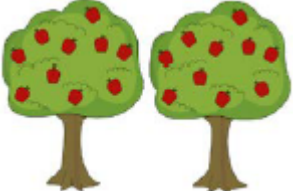
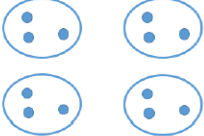

Children to count in 2s, 5s or 10s when counting their total number in the array.



$2 + 2 + 2 + 2 + 2 = 10$   
 $10 = 5 + 5$

Band 2

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

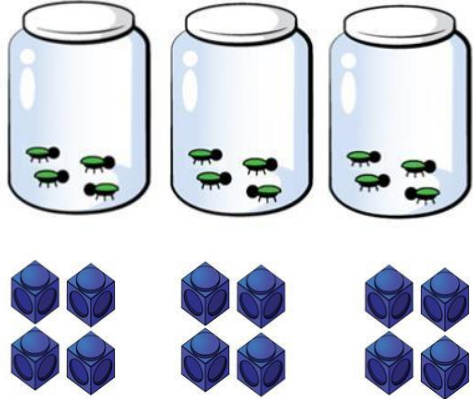
Concrete	Pictorial	Abstract
<p><b>Recognising equal groups</b> Use a range of visuals to support understanding of equal groups and making equal groups</p>  <p>3 equal groups of 2</p> 	<p>2 equal groups of 10</p>  	 <p>There are ___ equal groups with ___ in each group. There are three ____.</p> <p>___ + ___ + ___ = 12</p>

**Recognising the relationship between multiplication and repeated addition**

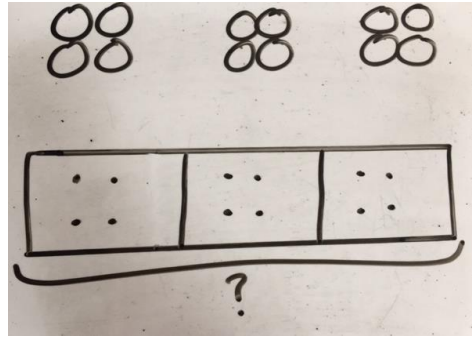
$$3 \times 4$$

$$4 + 4 + 4$$

There are 3 equal groups, with 4 in each group.



Children to represent the practical resources in a picture and use a bar model.



$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

**Create arrays** using children/counters/ cubes to show repeated addition, multiplication and commutativity.



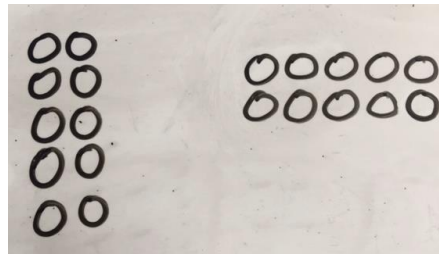
4 rows of 5 is 20

$$4 \times 5 = 20$$

5 rows of 4 is 20

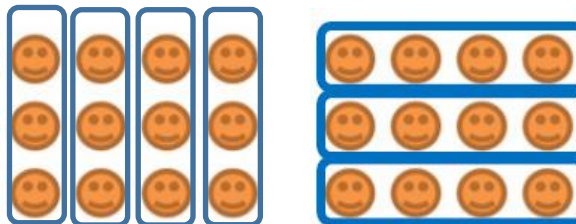
$$5 \times 4 = 20$$

Draw arrays in different rotations to find **commutative** multiplication sentences.



$$4 \times 3 = 12$$

$$3 \times 4 = 12$$



Children to be able to use an array to write a range of calculations e.g.

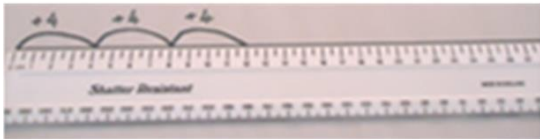
$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

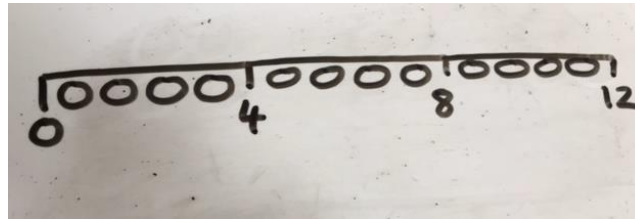
$$10 = 5 + 5$$

Number lines to show repeated groups-  $3 \times 4$



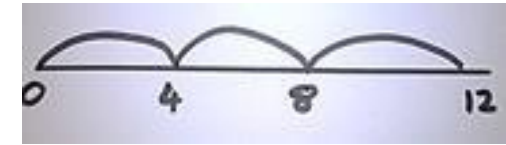
Cuisenaire rods can be used too.

Represent this pictorially alongside a number line e.g.:

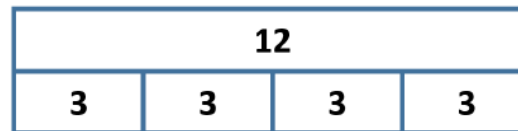


Abstract number line showing three jumps of four.

$$3 \times 4 = 12$$



Use bar model to show relationship between whole/parts for multiplication and makes links to division.



$$4 \times 3 = 12$$

$$12 \div 3 = 4$$

$$12 \div 4 = 3$$

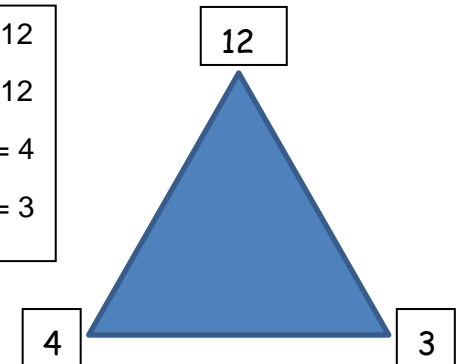
Use of the triangle to identify the 4 linked number sentences.

$$4 \times 3 = 12$$

$$3 \times 4 = 12$$


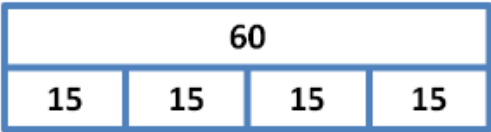
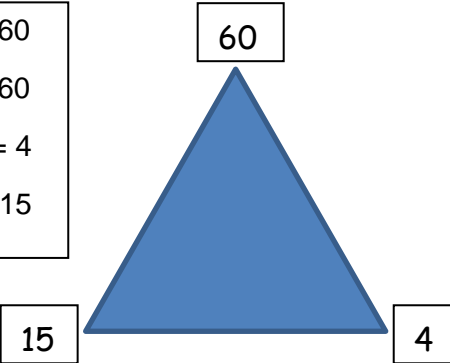
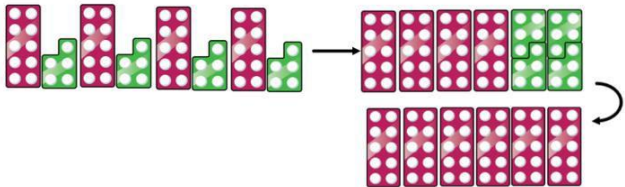
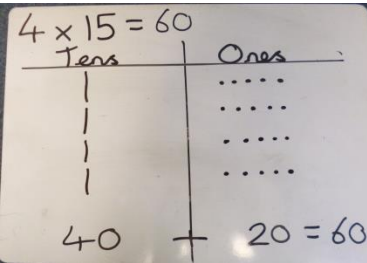
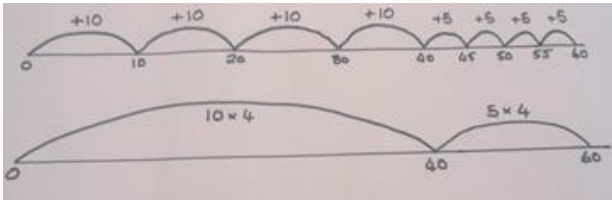
$$12 \div 3 = 4$$

$$12 \div 4 = 3$$



Band 3

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

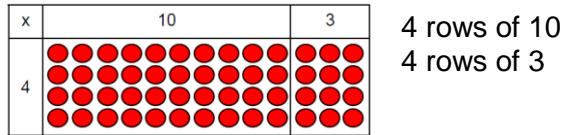
Concrete	Pictorial	Abstract
<p>Concrete resources to solve the number sentence, e.g. numicon, dienes, place value counters,  <math>4 \times 15 = 60</math></p> 	<p>Use bar model to show relationship between whole/parts for multiplication and makes links to division.</p>  <p><math>4 \times 15 = 60</math>  <math>60 \div 15 = 4</math>  <math>60 \div 4 = 15</math></p>	<p>Use of the triangle to identify the 4 linked number sentences.</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>15 \times 4 = 60</math>  <math>4 \times 15 = 60</math>  <math>60 \div 15 = 4</math>  <math>60 \div 4 = 15</math> </div> 
<p><b>Partition to multiply</b>            Partition to multiply with place value counters (dienes, numicon can also be used.) <math>3 \times 23</math>            Multiply the parts knowing known multiplication facts.            2 colours show distributive law.  <math>4 \times 15 = 60</math></p> 	<p><math>4 \times 15 = 60</math></p> 	<p>A blank number line can be drawn.</p> <p><math>4 \times 15 = 60</math></p> 



**Grid method**

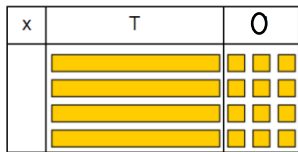
Show the link with arrays to first introduce the grid method.

$4 \times 13 = 52$



Move on to using dienes to move towards a more compact method.

4 rows of 13



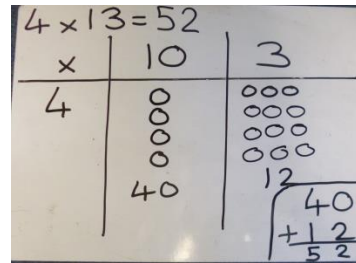
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

$4 \times 13 = 52$



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	10	3
4	40	12

$40 + 12 = 52$

**Formal expanded written method**

$$\begin{array}{r}
 13 \\
 \times 4 \\
 \hline
 12 = 4 \times 3 \\
 40 = 4 \times 10 \\
 \hline
 52 = 12 + 40
 \end{array}$$

Band 4


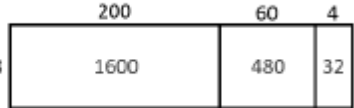
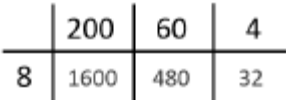
Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Concrete	Pictorial	Abstract																							
<p><b>Grid method</b> Use place value counters or dienes to support understanding, as seen in Band 3.</p>	<p><math>264 \times 8 = 2112</math></p> <table border="1" data-bbox="965 695 1254 799"><tr><td></td><td>200</td><td>60</td><td>4</td></tr><tr><td>8</td><td>1600</td><td>480</td><td>32</td></tr></table>		200	60	4	8	1600	480	32	<table data-bbox="1554 584 1671 735"><tr><td>5</td><td>3</td></tr><tr><td>2</td><td>6</td><td>4</td></tr><tr><td></td><td>x</td><td>8</td></tr><tr><td colspan="3"><hr/></td></tr><tr><td>2</td><td>1</td><td>1</td><td>2</td></tr></table>	5	3	2	6	4		x	8	<hr/>			2	1	1	2
	200	60	4																						
8	1600	480	32																						
5	3																								
2	6	4																							
	x	8																							
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2	1	1	2																						

Band 5

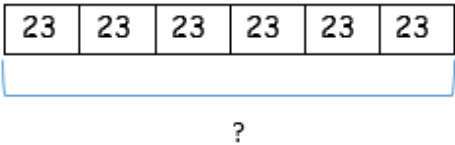
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Concrete	Pictorial	Abstract									
<p><b>Grid method</b> Use place counters to show how we are finding groups of a number. We are multiplying by 13, so we need 13 rows.</p> <p><math>18 \times 13 = 234</math></p> 	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p> <p><math>18 \times 13 = 234</math></p> <table border="1" data-bbox="882 1011 1261 1166"> <tr> <td>x</td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> <p><math>130 + 104 = 234</math></p>	x	10	8	10	100	80	3	30	24	<p><b>Formal written method</b> Expanded written method</p> $\begin{array}{r} 18 \\ \times 13 \\ \hline 24 = 8 \times 3 \\ 30 = 3 \times 10 \\ 80 = 10 \times 8 \\ \hline 100 = 10 \times 10 \\ \hline 234 = 24 + 30 + 80 + 100 \end{array}$ <p>Short column multiplication</p> $\begin{array}{r} 18 \\ \times 13 \\ \hline 234 \\ 2 \end{array}$
x	10	8									
10	100	80									
3	30	24									
<p><math>264 \times 8</math></p> 		$\begin{array}{r} 264 \\ \times 8 \\ \hline 2112 \end{array}$									

Band 6

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

## Conceptual variation; different ways to ask children to solve $6 \times 23$

 <p style="text-align: center;">?</p>	<p>Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p>With the counters, prove that <math>6 \times 23 = 138</math></p>	<p>Find the product of 6 and 23 <math>6 \times 23 =</math></p> <p style="margin-left: 40px;"><math>= 6 \times 23</math></p> <table style="margin-left: 40px;"> <tr> <td style="text-align: right;">6</td> <td style="text-align: right;">23</td> </tr> <tr> <td style="text-align: right;"><math>\times \underline{23}</math></td> <td style="text-align: right;"><math>\times \underline{6}</math></td> </tr> <tr> <td style="text-align: right;">——</td> <td style="text-align: right;">——</td> </tr> </table>	6	23	$\times \underline{23}$	$\times \underline{6}$	——	——	<p>What is the calculation? What is the product?</p>
6	23								
$\times \underline{23}$	$\times \underline{6}$								
——	——								